



NPTEL ONLINE CERTIFICATION COURSES

FINITE ELEMENT METHOD

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Module 04: FEM for Euler Bernoulli's Beam

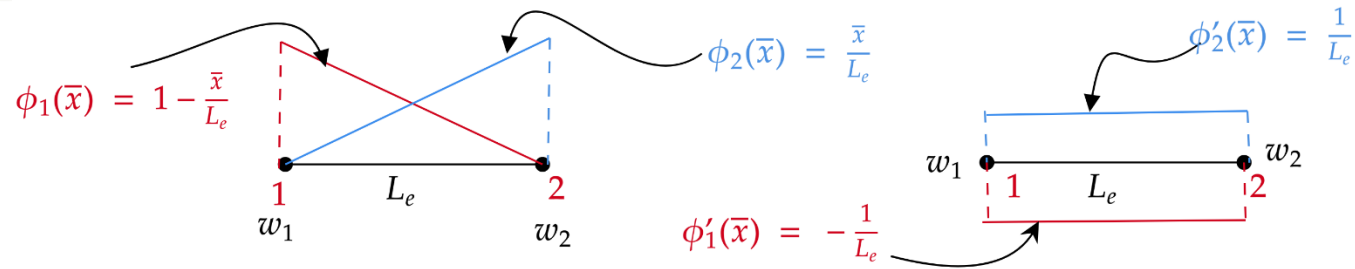
Lecture 17 : Beam Formulations

Concepts Covered

- Beam Equations
- Weak Form
- Hermite Polynomial

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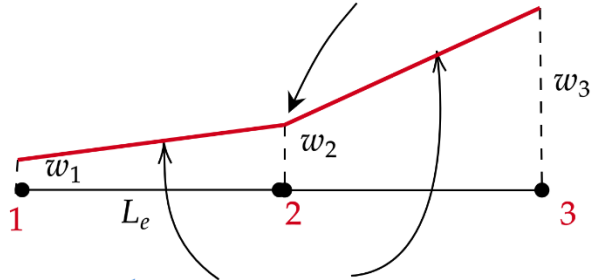
Linear Shape Function



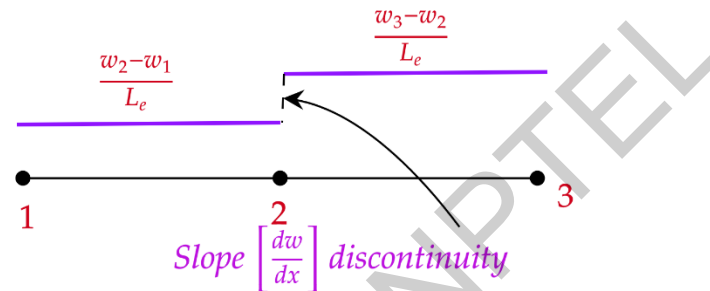
$$w(\bar{x}) = \phi_1(\bar{x})w_1 + \phi_2(\bar{x})w_2$$

$$\begin{aligned} \frac{dw(\bar{x})}{dx} &= \frac{d\phi_1(\bar{x})}{dx}w_1 + \frac{d\phi_2(\bar{x})}{dx}w_2 = \phi_1'(\bar{x})w_1 + \phi_2'(\bar{x})w_2 \\ &= \frac{w_2 - w_1}{L_e} \text{ (Constant)} \end{aligned}$$

$w(x)$ is C^0 continuous (at interelement boundary)



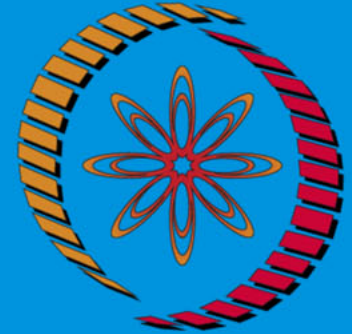
$w(x)$ is C^1 continuous (within the element)



Slope $\left[\frac{dw}{dx} \right]$ discontinuity
 (at interelement boundary, i.e. C^{-1})
 within elements C^0 continuous slope

$w(x)$ is C^1 continuous function within the element but C^0 continuous at interelement boundary
 $\frac{dw(x)}{dx}$ is C^0 continuous function within the element but discontinuous or C^{-1} at the interelement boundary

So, this element is C^0 element because it can give at least C^0 global continuity of the field variable



**THANK
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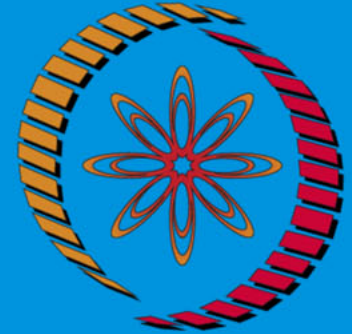
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Lecture 18 : Beam Stiffness Matrix

Concepts Covered

- Hermite Shape Function
- Element Stiffness Matrix
- Eigen Value Problem
- Mass Matrix

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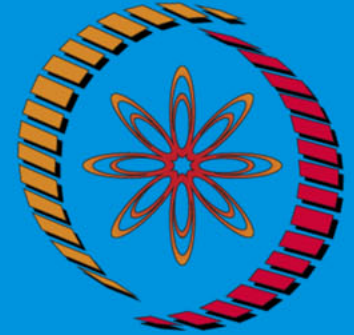
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Lecture 19 : Problems

Concepts Covered

- Problems

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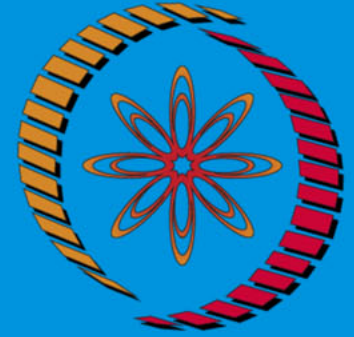
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Lecture 20 : Beam Column

Concepts Covered

- Beam Column – Potential Energy
- Euler Buckling Load
- Geometric Stiffness Matrix
- Problem



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Lecture 21 : Problem

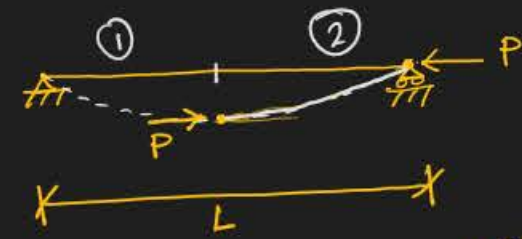
Concepts Covered

- Problem

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Geometric stiffness matrix

$$\frac{d^2}{dx^2} \left(EI \frac{d^2 w}{dx^2} \right) + P_{cr} \frac{d^2 w}{dx^2} = 0 \quad \text{and BC.}$$



$$w(0) = 0$$

$$EI \frac{d^2 w}{dx^2} \bigg|_{x=0} = 0$$

$$w(L) = 0$$

$$EI \frac{d^2 w}{dx^2} \bigg|_{x=L} = 0$$

$$w(0) = 0$$

$$EI \frac{d^2 w}{dx^2} \bigg|_{x=0} = 0$$

$$\frac{dw}{dx} \bigg|_{x=L/2} = 0$$

$$P_{cr, exact} = \frac{\pi^2 EI}{L^2}$$

$$= 9.87 \frac{EI}{L^2}$$

$$([K^e] - P_{cr} [K^g]) \{w\} = \{Q\} = \{0\}$$

$$K^e = \frac{2EI}{L^3} \begin{bmatrix} 6 & -3L & -6 & -3L \\ -3L & 2L^2 & 3L & L^2 \\ -6 & 3L & 6 & 3L \\ -3L & L^2 & 3L & 2L^2 \end{bmatrix}$$

$$K^g = \frac{1}{30L} \begin{bmatrix} 36 & -3L & -36 & -3L \\ -3L & 4L^2 & 3L & -L^2 \\ -36 & 3L & 36 & 3L \\ -3L & -L^2 & 3L & 4L^2 \end{bmatrix}$$

$$\{w\} = \begin{Bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{Bmatrix}$$

$$K_{ij} = \int_0^L \frac{d\phi_i}{dx} \frac{d\phi_j}{dx} dx$$

$$\det \left(\frac{2EI}{L^3} \begin{bmatrix} 2L^2 & 3L \\ 3L & 6 \end{bmatrix} - \frac{P_{cr}}{30L} \begin{bmatrix} 4L^2 & 3L \\ 3L & 36 \end{bmatrix} \right) = 0$$

$$45\lambda^2 - 26\lambda + 1 = 0$$

$$\lambda_{1,2} = \frac{13 \pm \sqrt{124}}{45}$$

$$\lambda = \frac{L^2}{60EI} P_{cr} \Rightarrow P_{cr} = 9.9439 \frac{EI}{L^2}$$

$$\lambda = \frac{L^2}{60EI} P_{cr}$$

$$\lambda_1 = \frac{13 - \sqrt{124}}{45} = 0.04133$$

$$P_{cr, exact} = 9.87 \frac{EI}{L^2}$$

_____ x _____

$$\frac{d^2}{dx^2} \left(EI \frac{d^2 w}{dx^2} \right) + P \frac{d^2 w}{dx^2} = q(x)$$

Free Vibration: —

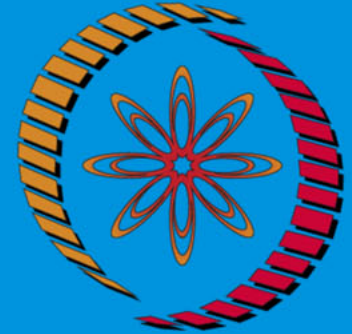


P is given here.

$$\frac{d^2}{dx^2} \left(EI \frac{d^2 w}{dx^2} \right) + P \frac{d^2 w}{dx^2} = \rho A \omega^2 w$$

$$([K^e] - P[K^g] - \omega^2 [M^e]) \{w\} = \{0\}$$

$$[M^e] = \frac{\rho A L}{420} \begin{bmatrix} 156 & -22L & 54 & 13L \\ -22L & 4L^2 & -13L & -3L^2 \\ 54 & -13L & 156 & 22L \\ 13L & -3L^2 & 22L & 4L^2 \end{bmatrix}$$



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